Calcul des déformations des ressorts hélicoïdaux

Forces distribuées - Modèle encastré - libre

Torseur des forces de cohésion

$$\mathbf{r_q}(\chi) := \begin{pmatrix} x_0(\chi) \\ y_0(\chi) \\ z_0(\chi) \end{pmatrix} \qquad \mathbf{r_S}(\alpha') := \begin{pmatrix} x_0(\alpha') \\ y_0(\alpha') \\ z_0(\alpha') \end{pmatrix} \qquad \mathbf{r_v}(\alpha) := \begin{pmatrix} x_0(\alpha) \\ y_0(\alpha) \\ z_0(\alpha) \end{pmatrix}$$

Forces distribuées entre α' et ψ_q

$$\mathbf{q}(\chi) := \begin{pmatrix} q_x(\chi) & q_y(\chi) & q_z(\chi) \end{pmatrix}^T \qquad \mathbf{M_q}(\alpha') := \left[\int_{\alpha'}^{\psi_q} \left(\mathbf{r_q}(\chi) - \mathbf{r_S}(\alpha')\right) \times \mathbf{q}(\chi) \cdot \frac{R}{\cos(\beta)}\, d\chi\right] \cdot (\alpha' \le \psi_q)$$

$$\mathbf{M_q}(\alpha') := \begin{bmatrix} \displaystyle\int_{\alpha'}^{\psi_q} \left[\left(\mathbf{r_q}(\chi) - \mathbf{r_S}(\alpha')\right) \times \mathbf{q}(\chi)\right]_0 \cdot \frac{R}{\cos(\beta)}\, d\chi \\ \displaystyle\int_{\alpha'}^{\psi_q} \left[\left(\mathbf{r_q}(\chi) - \mathbf{r_S}(\alpha')\right) \times \mathbf{q}(\chi)\right]_1 \cdot \frac{R}{\cos(\beta)}\, d\chi \\ \displaystyle\int_{\alpha'}^{\psi_q} \left[\left(\mathbf{r_q}(\chi) - \mathbf{r_S}(\alpha')\right) \times \mathbf{q}(\chi)\right]_2 \cdot \frac{R}{\cos(\beta)}\, d\chi \end{bmatrix} \cdot (\alpha' \le \psi_q)$$

Sollicitations

$$\mathbf{e'_1}(\alpha', \beta) := \begin{pmatrix} -\cos(\beta)\cdot\sin(\alpha') & \cos(\beta)\cdot\cos(\alpha') & \sin(\beta) \end{pmatrix}^T \qquad \mathbf{e'_2}(\alpha', \beta) := \begin{pmatrix} -\cos(\alpha') & -\sin(\alpha') & 0 \end{pmatrix}^T$$

$$\mathbf{e'_3}(\alpha', \beta) := \begin{pmatrix} \sin(\beta)\cdot\sin(\alpha') & -\sin(\beta)\cdot\cos(\alpha') & \cos(\beta) \end{pmatrix}^T$$

Moments de torsion $\quad M_{qt}(\alpha') := \mathbf{M_q}(\alpha')\cdot\mathbf{e'_1}(\alpha', \beta)$

Moments de flexion $\quad M_{qf2}(\alpha') := \mathbf{M_q}(\alpha')\cdot\mathbf{e'_2}(\alpha', \beta)$

$$M_{qf3}(\alpha') := \mathbf{M_q}(\alpha')\cdot\mathbf{e'_3}(\alpha', \beta)$$

Contraintes

k = limite d'élasticité en traction / limite d'élasticité en compression

M=1 point intérieur de la section droite sur l'axe O**e'**$_2$ M=3 point extérieur de la section droite sur l'axe O**e'**$_2$

M=2 point inférieur de la section droite sur l'axe O**e'**$_3$ M=4 point supérieur de la section droite sur l'axe O**e'**$_3$

$$\tau_q(\alpha', M) := \frac{M_{qt}(\alpha')}{W_t}\cdot[(M = 1) - 1\cdot(M = 3)] + \frac{M_{qt}(\alpha')}{W'_t}\cdot[(M = 4) - 1\cdot(M = 2)]$$

$$\sigma_q(\alpha', M) := \frac{M_{qf3}(\alpha')}{W_{f3}}\cdot[(M = 3) - 1\cdot(M = 1)] + \frac{M_{qf2}(\alpha')}{W_{f2}}\cdot[(M = 2) - 1\cdot(M = 4)]$$

$$\sigma_{\acute{e}quiv}(k, \alpha', M) := \frac{1 - k}{2}\cdot\left|\sigma_q(\alpha', M)\right| + \frac{1 + k}{2}\cdot\sqrt{\sigma_q(\alpha', M)^2 + 4\cdot\tau_q(\alpha', M)^2}$$

Calcul des déplacements par les intégrales de Mohr

Calcul des déplacements linéiques d'un point du ressort

Force unitaire virtuelle $\mathbf{v}(\lambda,\gamma) := (\cos(\lambda)\cdot\sin(\gamma) \quad \sin(\lambda)\cdot\sin(\gamma) \quad \cos(\gamma))^T$

Sollicitations dues à la force unitaire

$$\mathbf{M_v}(\alpha,\lambda,\gamma,\alpha') := \left[\left(\mathbf{r_v}(\alpha)-\mathbf{r_S}(\alpha')\right)\times\mathbf{v}(\lambda,\gamma)\right]\cdot(\alpha'<\alpha) \qquad M_{tv}(\alpha,\lambda,\gamma,\alpha') := \mathbf{M_v}(\alpha,\lambda,\gamma,\alpha')\cdot\mathbf{e'_1}(\alpha',\beta)$$

$$M_{fv2}(\alpha,\lambda,\gamma,\alpha') := \mathbf{M_v}(\alpha,\lambda,\gamma,\alpha')\cdot\mathbf{e'_2}(\alpha',\beta) \qquad M_{fv3}(\alpha,\lambda,\gamma,\alpha') := \mathbf{M_v}(\alpha,\lambda,\gamma,\alpha')\cdot\mathbf{e'_3}(\alpha',\beta)$$

Déplacement dans la direction de **v**

$$\delta_{qtv}(\alpha,\lambda,\gamma) := \frac{1}{G\cdot J_t}\cdot\int_0^{\alpha} M_{qt}(\alpha')\cdot M_{tv}(\alpha,\lambda,\gamma,\alpha')\cdot\frac{R}{\cos(\beta)}\,d\alpha'$$

$$\delta_{qfv2}(\alpha,\lambda,\gamma) := \frac{1}{E\cdot I_{22}}\cdot\int_0^{\alpha} M_{qf2}(\alpha')\cdot M_{fv2}(\alpha,\lambda,\gamma,\alpha')\cdot\frac{R}{\cos(\beta)}\,d\alpha'$$

$$\delta_{qfv3}(\alpha,\lambda,\gamma) := \frac{1}{E\cdot I_{33}}\cdot\int_0^{\alpha} M_{qf3}(\alpha')\cdot M_{fv3}(\alpha,\lambda,\gamma,\alpha')\cdot\frac{R}{\cos(\beta)}\,d\alpha'$$

$$\delta_{qv}(\alpha,\lambda,\gamma) := \delta_{qtv}(\alpha,\lambda,\gamma)+\delta_{qfv2}(\alpha,\lambda,\gamma)+\delta_{qfv3}(\alpha,\lambda,\gamma)$$

Calcul des déplacements angulaires

Couple unitaire virtuel $\mathbf{cv}(\lambda_c,\gamma_c) := \left(\cos(\lambda_c)\cdot\sin(\gamma_c) \quad \sin(\lambda_c)\cdot\sin(\gamma_c) \quad \cos(\gamma_c)\right)^T$

Sollicitations dues au couple unitaire

$$\mathbf{M_{cv}}(\alpha,\lambda_c,\gamma_c,\alpha') := \mathbf{cv}(\lambda_c,\gamma_c)\cdot(\alpha'<\alpha)$$

$$M_{tcv}(\alpha,\lambda_c,\gamma_c,\alpha') := \mathbf{M_{cv}}(\alpha,\lambda_c,\gamma_c,\alpha')\cdot\mathbf{e'_1}(\alpha',\beta)$$

$$M_{fcv2}(\alpha,\lambda_c,\gamma_c,\alpha') := \mathbf{M_{cv}}(\alpha,\lambda_c,\gamma_c,\alpha')\cdot\mathbf{e'_2}(\alpha',\beta) \qquad M_{fcv3}(\alpha,\lambda_c,\gamma_c,\alpha') := \mathbf{M_{cv}}(\alpha,\lambda_c,\gamma_c,\alpha')\cdot\mathbf{e'_3}(\alpha',\beta)$$

Déplacement angulaire autour de l'axe défini par **cv**

$$\theta_{qtcv}(\alpha,\lambda_c,\gamma_c) := \frac{1}{G\cdot J_t}\cdot\int_0^{\alpha} M_{qt}(\alpha')\cdot M_{tcv}(\alpha,\lambda_c,\gamma_c,\alpha')\cdot\frac{R}{\cos(\beta)}\,d\alpha'$$

$$\theta_{qfcv2}(\alpha,\lambda_c,\gamma_c) := \frac{1}{E\cdot I_{22}}\cdot\int_0^{\alpha} M_{qf2}(\alpha')\cdot M_{fcv2}(\alpha,\lambda_c,\gamma_c,\alpha')\cdot\frac{R}{\cos(\beta)}\,d\alpha'$$

$$\theta_{qfcv3}(\alpha,\lambda_c,\gamma_c) := \frac{1}{E\cdot I_{33}}\cdot\int_0^{\alpha} M_{qf3}(\alpha')\cdot M_{fcv3}(\alpha,\lambda_c,\gamma_c,\alpha')\cdot\frac{R}{\cos(\beta)}\,d\alpha'$$

$$\theta_{qcv}(\alpha,\lambda_c,\gamma_c) := \theta_{qtcv}(\alpha,\lambda_c,\gamma_c)+\theta_{qfcv2}(\alpha,\lambda_c,\gamma_c)+\theta_{qfcv3}(\alpha,\lambda_c,\gamma_c)$$